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GENETIC ALGORITHM APPROACH FOR BANDPASS PROBLEM

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Abstract: *Bandpass Problem* (BP) is a telecommunication problem. This problem arises in considering the optimal packing of information flows on different wavelengths into groups to obtain the highest available cost reduction in design and operating the optical communication networks using wavelength division multiplexing technology.

Given a rectangular matrix A of binary elements $\{0, 1\}$ and a positive integer B called the *Bandpass Number*, a set of B consecutive non-zero elements in any column is called a Bandpass. No two bandpasses in the same column can have common rows.

The Bandpass problem consists of finding an optimal permutation of rows of the matrix, which produces the maximum total number of bandpasses having the same given bandpass number in all columns.

This combinatorial problem arises in considering the optimal packing of information flows on different wavelengths into groups to obtain the highest available cost reduction in design and operating the optical communication networks using wavelength division multiplexing technology.

Bandpass problem is in NP-hard class. Therefore, in this paper, a meta-heuristic method, Genetic Algorithms (GA), which are close to optimal solution but do not always find optimal solution, are formed. This GAs have been tested on bandpass library problems (Babayev *et al.*, 2007a) and the results are discussed.

Keywords: bandpass problem, combinatorial optimization, np-hard problem, genetic algorithm, artificial intelligence, meta-heuristic algorithm.

1. Introduction

A combinatorial optimization problem, called the *Bandpass Problem,* is often encountered in managing telecommunications networks and arises in considering the optimal packing of information flows on different wavelengths into groups to obtain the highest available cost reduction in design and operating the optical communication networks using wavelength division multiplexing technology.

BP was first raised by Bell and Babayev in 2004 (Bell and Babayev, 2004). Nuriyev and Kutucu (2007) created Boolean Integer, combinatorial and graph mathematical models (Nuriyev and Kutucu, 2007). Babayev, Nuriyev and Berberler (2007) worked on the complexity analysis of BP (Babayev *et al.*, 2007b).

Nuriyev, Gürsoy and Berberler (2007) developed a heuristic algorithm to solve BP in polinomial time (Nuriyev *et al.*, 2007). Babayev, Nuriyev, Bell, Kurt, Berberler and Gürsoy (2007) established an online Bandpass Problems library in order to compare the efficiency of the developed algorithms (Babayev *et al.*, 2007a). Problem instances of this library are divided into two classes whose optimal solutions are known and unknown. Each of these classes has 45 samples.

Babayev, Bell and Nuriyev (2009) proved that a combinatorial optimization problem called bandpass problem was NP-hard (Babayev *et al.*, 2009). Lin G. (2009), proved NP-Hardness of BP within a different way and developed polinomial algorithms for special cases (Lin, 2009).

In this paper, four genetic algorithm approaches, which were first created for BP, are introduced. Solutions of examples of the BP were developed with the genetic algorithms combined with a pre-established heuristic algorithm (Nuriyev *et al.*, 2007). It was observed that solutions of many problems were improved with computational experiments.

Problem Instances used in computational experiments were taken from the online bandpass problem library and the optimal solution values of these problems are known (Babayev *et al.*, 2007a).

2. Genetic Algorithms

Genetic algorithms are stochastic search algorithms based on the mechanism of natural selection and natural genetics. Genetic algorithms, in contrast to conventional search techniques, start with an initial set of random solutions called the population. Each individual in the population is called a chromosome, encoding a solution to the problem at hand. A chromosome is a string of symbols, usually but not necessarily, a binary bit string. Each element of this string is called gene (Fig. 2.1). With a combination of these chromosomes are creating the next generations.

Chromosome =	**Gene1**	**Gene2**	**Gene3**	**Gene4**	**…**	**GeneM**

Population = {Chromosome1, Chromosome2,…, ChromosomeN}

Fig. 2.1

The chromosomes evolve through successive iterations, called generations. During each generation, the chromosomes are evaluated, using some measures of fitness to create the next generation, new chromosomes, called offspring, are formed by either merging two chromosomes from the current generation using a crossover operator or modifying a chromosome using a mutation operator. A new generation is formed by selecting, according to the fitness values, some of the parents and offspring, and rejecting others so as to keep the population size constant. Fitter chromosomes have higher probabilities of being selected. After several generations, the algorithms converge to the best chromosome, which we hope represents the optimum or suboptimal solution to the problem when decoded The GA to find the optimal result, or until a predetermined number of iterations or until they reach a certain fitness value will continue to work.

GA can be used in constrained and unconstrained optimization problems. For example, non-linear, stochastic, Traveling Salesman, Knapsack Problem, Minimum Spanning Tree, and such scheduling problems can be applied to many combinatorial optimization problems (Gen, 2006).

3. Bandpass Problem

BP was the first time presented and formulated in Annual INFORMS meeting, October 2004, Denver, CO, USA. Today's Dense Wavelength Division Multiplexing (DWDM) technology, the fiber optic cables can carry information coded in different wavelengths. DWDM technology can carry up to 160 different wavelengths. While the information is moving within wavelengths, it is extracted from the fiber optic cable between the stations with Add/Drop Multiplexers (ADM) devices. In each ADM, there are special cards that control each wavelength and the cost of these special cards is very high. Several flows moved in different wave lengths often pass through the same ADM (Fig. 3.1). If these wavelengths are sequential, then they can be grouped under the same wavelength – bandpasses. In this case each card serves a specific number, called bandpass number B, of flows. This allows replacing several (B) cards for the bandpass, one for each wavelength, by one single card. This decreases the total number of cards and leads to reduction of the total cost related to cards. Therefore, this creates a cost reduction opportunity, which is important to be used to improve performance parameters in design and operating of optical communication networks and gives rise to the problem of optimally assigning information packages to wavelengths, with the purpose of creating the maximum number of bandpasses having the same single bandpass number over all ADMs (Babayev *et al.* 2009; Bell and Babayev, 2004).

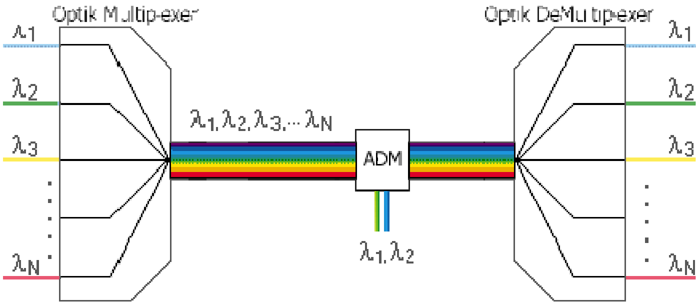


Fig. 3.1

There can be multiple stations in optical communication and an information package can be sent to multiple stations. If information package i = 1, ..., m is destined for point j = 1, ..., n, then a_{ij} = 1, otherwise a_{ij} = 0.

BP can be modeled mathematically as follows. Let A={aij}, a Boolean matrix, has m rows and n columns. B consecutive non-zero entries of matrix A in the same column form a Bandpass. Boolean integer Mathematical Model of the Bandpass Problem:

$$\text{Max} \sum_{j=1}^{n} \sum_{k=1}^{m-B+1} y_{kj}$$

$$\sum_{k=1}^{m} x_{ik} = 1, \quad i = \overline{1,m} \tag{1}$$

$$\sum_{i=1}^{m} x_{ik} = 1, \quad k = \overline{1,m} \tag{2}$$

$$\sum_{i=k}^{k+B-1} y_{ij} \le 1, \qquad j = \overline{1,n}, \quad k = \overline{1,(m-B+1)} \tag{3}$$

$$By_{kj} \le \sum_{i=k}^{k+B-1} \sum_{r=1}^{m} a_{rj} x_{rj}, \qquad j = \overline{1,n}, \quad k = \overline{1,(m-B+1)} \tag{4}$$

$$x_{ik} = \begin{cases} 1, \text{if row i is relocated to position k,} \\ 0, \text{otherwise.} \end{cases}, \quad i = \overline{1,m};\ k = \overline{1,m},$$

$$y_{kj} = \begin{cases} 1, \text{if row k is the first row of a bandpass in column j,} \\ 0, \text{otherwise.} \end{cases}, \quad j = \overline{1,n};\ k = \overline{1,m}.$$

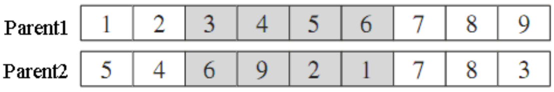
For example, let be m=96, n=25, B=16. In this case, main constraints respectively include 96, 96, 2025 and 2025, and this example totally includes 4226 constraints and 11241 variables. Optimal solution couldn't be found in reasonable time limits when computational experiments were performed with Lingo and NEOS solvers

4. Genetic Algorithm for Bandpass Problem

BP is structurally similar to the Traveling Salesman Problem TSP. The solution of the problem is a permutation of the input problem. It is very clear and easy to apply GA to TSP. Randomly generated chromosomes to be done in any decreasing distance intercity crossover and mutation methods can be directly affected.

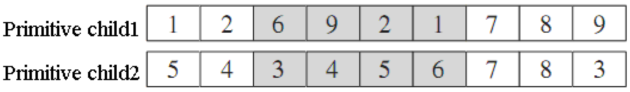
BP to GA adaptation is different from the TSP. The solution of the problem, however, a permutation of the problem must be entered as input, where the problem mXn size 0 and 1 is a matrix containing values. For the solution, m rows must be replaced so that new mXn dimensional matrix has the largest total number of bandpass. GA population to create using only randomly generated permutations row makes it difficult to obtain efficient solutions. At this point, creating a population for the GA to be obtained a solution from heuristic algorithm (Nuriyev *et al.*, 2007) without random row permutations is also effective to use. Heuristic algorithms to be obtained from the solutions are approximate solutions efficiently enough, or even lead to the GA and will be used to move away from inefficient again.

GA created the following crossover and mutation techniques are used. Crossover process with specific criteria in the two selected parent chromosomes are selected from the range of a gene (Fig. 4.1). After in the range of the selected gene is replaced with two chromosomes (Fig. 4.2).



Parent1	1	2	3	4	5	6	7	8	9
Parent2	5	4	6	9	2	1	7	8	3

Fig. 4.1. Parent chromosomes



Primitive child1	1	2	6	9	2	1	7	8	9
Primitive child2	5	4	3	4	5	6	7	8	3

Fig. 4.2. Primitive child chromosomes

After this change, primitive child chromosomes are obtained. Derived from primitive child chromosome, the gene can be encountered again. Since in this case, the selected genes are created from a range of permutations and primitive functions in children outside of this range is subject to the function of genes (Fig. 4.3). So again in the same chromosome the gene is eliminated.

After this procedure, two new child chromosomes are obtained (Fig. 4.4). According to the fitness value of GA operations is included for use in the population.

Fig. 4.3. Permutation function

Fig. 4.4. Child chromosomes

Two approaches were applied for the mutation process. The first is the two-point mutation. Where on a chromosome selected .according to desired criteria, randomly selected among the two genes are exchanged (Fig. 4.5 and Fig. 4.6). To increase the effectiveness of this method, with which the new chromosome is compared to parental chromosomes. The chromosome fitness value is greater in the general population continues to live while the other is destroyed. In the literature this method is referred to as 2-opt.

Fig. 4.5. Parent chromosome

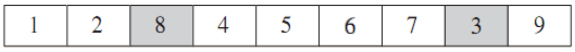


Fig. 4.6. New chromosome after two-point mutation

Other method used in this algorithm is three point mutations. In this method, a chromosome is chosen again according to desired criteria. Selected three different genes on a chromosome is selected (Fig. 4.7) and permutation of this gene with the 5 new chromosomes are obtained. Fitness value of chromosome or chromosomes with large populations continue to experience, while others are destroyed. In the literature this method is referred to as 3-opt.

Fig. 4.7. Parent chromosome

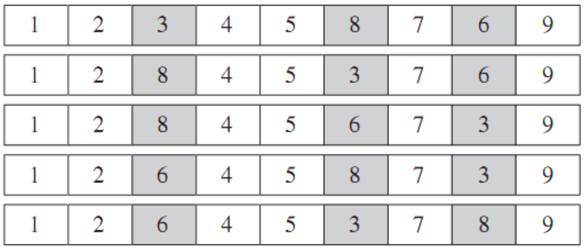


Fig. 4.8. Five new chromosomes after three-point mutation

This mutation and crossover techniques in the fuzzy GA mutation and crossover rates are used. The reason for this which will occur in GA appropriate solutions to avoid repeated and to increase the diversity of chromosomes.

5. Computational Experiments

GAs created, Bandpass Problem Library (Babayev *et al.*, 2007a) is working on the 45 problems. These 45 problems taken from the bandpass problem library, the optimal solutions of the problems are exactly known. Bandpass numbers obtained and their proximities to the optimum percentage are listed in Table 5.1.

In notation m is the number of rows, n is the number of columns, b is the width of bandpass and d, the density matrix as the ones used.

80% of the 45 problems of the calculation results of the experiment have been developed.

Unlike classic genetic algorithms, in these genetic algorithms, chromosomes are similar in population and decrease again to stay away from the two approaches was made. First, the next generation of parents and children to be transferred as a proportional change coefficients of chromosomes. Other, crossover and mutation rates in the variable structure are determined intervals. So you can change which results when re-entered again with the prevention of mutation and crossover rates are provided.

Table 5.1. Results of computational experiments

No	Problem	m_n_b_d	Opt	Heu	%	GA1	%	GA2	%	GA3	%	GA4	%
1	A28B5	64_8_5_25	20	19	0,950	19	0,950	19	0,950	19	0,950	**20**	1,000
2	A29B5	64_8_5_35	30	29	0,967	29	0,967	29	0,967	29	0,967	29	0,967
3	A30B8	64_8_8_25	12	11	0,917	**12**	1,000	**12**	1,000	11	0,917	11	0,917
4	A31B8	64_8_8_35	19	16	0,842	16	0,842	16	0,842	16	0,842	16	0,842
5	A32B16	64_8_16_25	10	6	0,600	6	0,600	6	0,600	6	0,600	6	0,600
6	A33B16	64_8_16_35	13	10	0,769	**11**	0,846	**11**	0,846	**11**	0,846	**11**	0,846
7	A34B5	64_12_5_25	30	27	0,900	27	0,900	27	0,900	27	0,900	27	0,900
8	A35B5	64_12_5_35	46	35	0,761	**37**	0,804	**37**	0,804	**37**	0,804	**37**	0,804
9	A36B8	64_12_8_25	23	18	0,783	**19**	0,826	**19**	0,826	**19**	0,826	**19**	0,826
10	A37B8	64_12_8_35	32	24	0,750	**24**	0,750	**24**	0,750	**24**	0,750	**24**	0,750
11	A38B16	64_12_16_25	17	7	0,412	**11**	0,647	10	0,588	**11**	0,647	**11**	0,647
12	A39B16	64_12_16_35	22	13	0,591	**14**	0,636	13	0,591	13	0,591	13	0,591
13	A40B5	64_16_5_25	44	35	0,795	36	0,818	**38**	0,864	36	0,818	35	0,795
14	A41B5	64_16_5_35	65	51	0,785	51	0,785	**52**	0,800	51	0,785	51	0,785
15	A42B8	64_16_8_25	33	22	0,667	**23**	0,697	22	0,667	22	0,667	22	0,667
16	A43B8	64_16_8_35	46	29	0,630	29	0,630	**31**	0,674	29	0,630	30	0,652
17	A44B16	64_16_16_25	24	12	0,500	**15**	0,625	13	0,542	**15**	0,625	**15**	0,625
18	A45B16	64_16_16_35	31	17	0,548	17	0,548	17	0,548	17	0,548	17	0,548
19	A46B8	96_8_8_25	19	16	0,842	**17**	0,895	16	0,842	16	0,842	16	0,842
20	A47B8	96_8_8_35	28	25	0,893	**25**	0,893	**25**	0,893	**25**	0,893	**25**	0,893
21	A48B16	96_8_16_25	11	9	0,818	9	0,818	9	0,818	9	0,818	9	0,818
22	A49B16	96_8_16_35	16	10	0,625	**12**	0,750	11	0,688	**12**	0,750	**12**	0,750
23	A50B8	96_16_8_25	42	28	0,667	28	0,667	28	0,667	28	0,667	28	0,667
24	A51B8	96_16_8_35	61	41	0,672	**44**	0,721	43	0,705	43	0,705	43	0,705
25	A52B16	96_16_16_25	29	17	0,586	**19**	0,655	**19**	0,655	17	0,586	**19**	0,655
26	A53B16	96_16_16_35	38	21	0,553	**23**	0,605	**23**	0,605	22	0,579	22	0,579
27	A54B8	96_25_8_25	73	48	0,658	50	0,685	**51**	0,699	50	0,685	49	0,671
28	A55B8	96_25_8_35	103	60	0,583	61	0,592	61	0,592	**62**	0,602	61	0,592
29	A56B16	96_25_16_25	49	25	0,510	**28**	0,571	**28**	0,571	25	0,510	26	0,531
30	A57B16	96_25_16_35	64	34	0,531	**34**	0,531	**34**	0,531	**34**	0,531	**34**	0,531
31	A58B16	64_8_16_75	23	19	0,826	19	0,826	**20**	0,870	19	0,826	19	0,826
32	A59B8	64_12_8_75	71	65	0,915	65	0,915	**66**	0,930	65	0,915	65	0,915
33	A60B5	64_16_5_50	96	76	0,792	76	0,792	**78**	0,813	76	0,792	**78**	0,813
34	A61B5	64_25_5_65	205	171	0,834	171	0,834	171	0,834	171	0,834	171	0,834
35	A62B8	64_25_8_65	137	104	0,759	**107**	0,781	**107**	0,781	**107**	0,781	**107**	0,781
36	A63B16	64_25_16_50	66	40	0,606	41	0,621	**42**	0,636	41	0,621	**42**	0,636
37	A64B16	96_8_16_65	30	24	0,800	24	0,800	**25**	0,833	24	0,800	24	0,800
38	A65B16	96_25_16_50	86	48	0,558	48	0,558	49	0,570	**53**	0,616	49	0,570
39	A66B5	96_8_5_75	110	**110**	1,000	**110**	1,000	**110**	1,000	**110**	1,000	**110**	1,000
40	A67B8	96_8_8_75	67	67	1,000	67	1,000	67	1,000	67	1,000	67	1,000
41	A68B16	96_16_16_65	67	49	0,731	**52**	0,776	49	0,731	49	0,731	49	0,731
42	A69B5	96_16_5_65	187	162	0,866	**165**	0,882	162	0,866	162	0,866	162	0,866
43	A70B25	96_16_25_25	25	11	0,440	11	0,440	11	0,440	11	0,440	11	0,440
44	A71B8	96_25_8_50	148	92	0,622	**94**	0,635	92	0,622	92	0,622	92	0,622
45	A72B25	96_25_25_35	50	25	0,500	28	0,560	28	0,560	25	0,500	**29**	0,580

Four GAs which are created with this approach are not very different from each other and these algorithms have good efficiency in general. Problems in general, as the average efficiency, heuristic algorithm has performed %72, GA1 has performed %75 and GA2, GA3, and GA4 have performed %74 efficiency.

6. Conclusion

In this paper, four new GAs are the first time created for Bandpass problem, an NP-hard combinatorial optimization problem with important applications. These GAs are supported by previously developed heuristic and results produced by this heuristic have been developed. It is observed that the GAs has efficient results.

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