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A FUZZY PERSPECTIVE FOR TWO-DIMENSIONAL PACKING OF VARIABLE-SIZED ITEMS

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Abstract: In this study, a two-dimensional packing problem encountered in the telecommunications area is investigated, which deals with the allocation of data transmission from a base station to many mobile stations for certain frames of time with varying objectives. The packed items have rectangular areas with variable width and height. Our objective is to maximize the overall quality of packing through a sequence of time frames. We use fuzzy area definitions to allow partial packing among frames, considering item priorities and service quality. We model the problem and discuss solution approaches.

Keywords: two-dimensional packing, fuzzy constraints, resource allocation, packing quality.

1. Introduction

WiMAX (Worldwide Interoperability for Microwave Access) is one of the broadband wireless access technologies, which is based on the IEEE 802.16 standard (2009). The aim of this technology is providing wireless data transfer using various transmission modes, including point-to-multipoint connections and portable or fully mobile cellular type access. A WiMAX base station (BS) can provide broadband wireless access in range up to 50 kms for fixed stations and 5 to 15 kms for mobile stations (MS) with a maximum data rate of up to 70 Mbps (IEEE 802.16-2009).

Apart from its technical qualities, three features of WiMAX mainly attract our attention. These are: (1) the use of Orthogonal Frequency Division Multiple Access (OFDMA), (2) multiple Quality of Service (QoS) classes that define priorities between data, voice and video transmissions for satisfying service guarantees, and (3) the so-called Media Access Control (MAC) scheduler of the BS which utilizes the first two aspects (So-In *et al.*, 2009a; Necker *et al.*, 2008).

OFDMA is based on Orthogonal Frequency Division Multiplexing (OFDM) which is a spread-spectrum technique for state-of-the-art broadband wireless systems. The frequency spectrum used for communications is divided into a large number of frequency subcarriers to serve different terminals in the same time intervals and through the same physical channels.

QoS classes allow the BS to classify the terminals according to parameters like minimum throughput requirements regarding data transmission rates and delay constraints. The QoS support in wireless network connections is much more demanding than that in wired networks due to its highly variable and unpredictable nature depending both on time and locations of the terminals.

The MAC scheduler is responsible for two tasks concerning resource allocation both of which are related to our model in this study. First one is to determine the terminals which shall be served in a specific time frame, thus forming a service queue, and also to determine the amount of data required for each terminal. This depends mainly on the QoS parameters used for that particular network. This task is studied under names like burst construction (Ohseki *et al.*, 2007) or packet scheduling (Wongthavarawat and Ganz, 2003). They present an uplink (transmission from MS to BS) packet scheduling algorithm and admission control policy to provide QoS support in terms of bandwidth and delay bounds for different traffic classes. They investigate the factors which affect the network performance by a simulation model.

The other decision of the scheduler is assigning time and frequency intervals to each terminal, referred to as frame packing, packet mapping or burst mapping in the literature (Ben-Shimol *et al.*, 2006; Bacioccola *et al.*, 2007; Ohseki *et al.*, 2007; Necker *et al.*, 2008; So-In *et al.*, 2009b). We will use the term "frame packing" henceforth in this paper.

IEEE 802.16 standard does not impose any specific admission control mechanisms or resource allocation mechanisms for the scheduler. Therefore, scheduling becomes a significant topic for all WiMAX equipment makers and network service providers. So-In *et al.* (2009a) gives an extensive review for the main issues considered in designing these mechanisms. They explain the physical layers and QoS classes defined in the IEEE 802.16 standard, and classify the schedulers based on channel state awareness. They list various criteria and scheduler design factors with respect to different QoS considerations which shape the algorithms and heuristics proposed in the literature. Throughput maximization and minimizing power consumption are among the major criteria while ensuring system scalability regarding algorithm complexity.

The frame to be packed is a two-dimensional structure defined in the IEEE 802.16 standard using OFDMA in which the frequency channel is divided into multiple subcarriers. These subcarriers are grouped into a number of subchannels. The time axis of the frame typically covers a 5ms duration. Each user terminal is allocated a certain number of subchannels for a certain amount of time. Bidirectional data transfer can be achieved in two ways, either by frequency division duplexing (FDD) in which uplink and downlink (BS-to-MS direction) use different frequency bands or by time division duplexing (TDD) in which the uplink (UL) traffic follows the downlink (DL) traffic in time dimension (So-In *et al.*, 2009a). Hence, each frame consists of DL and UL subframes. Fig. 1 shows these in TDD mode. In FDD mode, DL and UL subframes go parallel in time. The proposed model in our study can be adapted for both FDD and TDD modes, so from now on we will focus on TDD systems, and in particular on packing DL subframe.

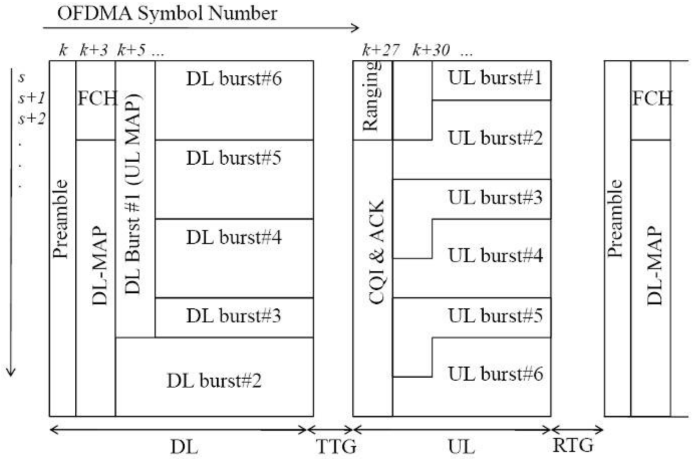

Fig. 1. A sample OFDMA frame structure in TDD mode (Source: So-In *et al.*, 2009b, pp. 1)

User terminals are mapped to rectangular areas (referred to as bursts) in the OFDMA frame. The unit of allocation in WiMAX is “slot”. The data transmission capacity of a slot definition depends upon the subchannelization or modulation and coding methods which are not the subjects of our study. Rather, we base our problem on the premise that the IEEE 802.16 standard dictates all DL data bursts to be rectangular. Moreover, we consider a one-to-one mapping from each terminal or MS to a DL burst.

The standard allows more than one burst per mobile station, which increases the downlink map (DL-MAP) overhead (amount of control data). The DL-MAP and uplink map (UL-MAP) define the burst-start time and burst-end time, modulation types and forward error control mechanisms for each MS. Let us summarize the other terms and parts of the frame shown in Fig. 1: The preambles are used for time synchronization. Frame Control Header (FCH) defines MAP lengths and usable subcarriers. Transmit-receive Transition Gap (TTG) is the duration that the BS switches from transmit to receive mode, and Receive-transmit Transition Gap (RTG) occurs when the BS switches from receive to transmit mode. Channel Quality Indicator (CQI) is used to pass the channel state condition information. Each OFDM symbol is 102.8 μs, so a 5ms frame is equal to a duration of 48.6 OFDM symbols. (So-In *et al.*, 2009a; So-In *et al.*, 2009b). The standard also allows more than one connections packing into one burst (burst compaction). However, in each case the MAC scheduler is required to place rectangles in the frame as efficiently as possible under technology and traffic specific constraints.

Ben-Shimol *et al.* (2006) were first to introduce an OFDMA frame packing algorithm. They present two heuristics, without and with QoS constraints, which were evaluated by extensive simulations using the parameters of real systems. They place the data bursts row by row with a non-increasing size order. Ohseki *et al.* (2007) propose a burst construction and frame packing method in the DL subframe. Their objective is not only to decrease the control data ratio within the rectangles, but also control data that must be transmitted at the head of every frame, to reach higher throughput. They define deadlines for each connection using QoS parameters and order them according to remaining time to deadlines.

As the number of bursts to be packed in a frame increases, so does the running time of the packing solution. Similarly, when a single terminal connection is mapped into multiple rectangular areas (Bacioccola *et al.*, 2007), additional DL-MAP overhead causes inefficient usage of network resources. This particular case may be effective when the connection really requires different reliable channels using different modulation and coding schemes for different connections, despite the additional cost incurred. In the burst compaction case where multiple terminals are packed into a single burst, the unique connection identifier (CID) helps differentiate the terminals. The DL-MAP overhead is reduced, but may lead to decoding delay at the terminal side.

Necker *et al.* (2008) propose a genetic algorithm for the DL frame packing problem. Simulating the network traffic with three different QoS classes, they apply the Next-Fit-Decreasing-Height (NFDH) strip packing approach to maximize DL capacity utilization. In strip packing, the objective is to pack all items within the minimum height. They base their genome modeling on the variable width and height dimension sizes of the DL bursts, since there are many possible height and width combinations corresponding to a burst area.

So-In *et al.* (2009b) also introduce an algorithm for the DL frame packing. They approach the MAC resource scheduling in two steps. First, the scheduler sorts each user terminal in a descending order based on their demands for satisfying the QoS throughput guarantee. Then, the bursts are packed from right to left and from bottom to top in the DL subframe. The authors choose the burst shape that is smallest in width to allow the receiving MS to shut down its electronic circuit for the remainder of the DL subframe, thus minimizing energy consumption.

Lodi *et al.* (2002) review solution approaches for the general two-dimensional packing problems. They list the heuristics and exact algorithms in the literature both for bin packing where the objective is to pack all the items into the minimum number of units, and for strip packing. In another noteworthy study, Martello and Vigo (1998) propose new lower bounds which are used within an exact branch-and-bound algorithm by investigating a well known lower bound and determining its worst-case performance. Kenyon and Rémila (2000) present an approximation scheme for two-dimensional strip packing problem based on a new linear programming relaxation. For any given ε, their algorithm finds a feasible solution within a factor of $(1+\varepsilon)$ of the optimum with a polynomial running time complexity both in number of items and in $1/\varepsilon$. Caprara and Monaci (2004) on the other hand, deal with the two-dimensional knapsack problem (2KP), aimed at packing a maximum-profit subset of rectangles. They consider the natural relaxation of 2KP given by the one-dimensional KP with item weights equal to the rectangle areas. They present four exact algorithms based on that relaxation, proving the worst-case performance of the associated upper bound, and computationally compare them.

2. Problem Definition

The frame packing solution approaches summarized in the previous section do not allow partial allocations, as typical bin packing algorithms in the literature. Hence, an item must be placed fully in a bin or cannot be placed at all. However, as in the wireless communication systems, some service resource allocation problems may possess highly variable and unpredictable characteristics. The service demand levels may not be exactly satisfied in some periods of time, or with respect to some other criteria.

Kim *et al.* (2001) introduce the fuzzy bin packing problem defined as packing non-rigid rectangular items into an open rectangular bin, thus actually a strip packing problem. They employ fuzziness in the height dimension by using triangular fuzzy numbers. Their aim is to minimize both the height of a packing and the extra cost due to the size reduction of each item. The authors present a closed form solution by representing the total cost as the sum of the height cost and the size reduction cost given by a quadratic function. Reducing the height of an item decreases the overall height cost but increases the reduction cost due to lower quality of the item.

Nasibov (2004) proposes a new approach for the bin packing problem with the evaluation of the packing quality under fuzzy source constraints. The items are to be allocated in two sets of containers, one is comprised by m main containers S_i and the second has only one reserve container S_{m+1}. The author defines four fuzzy relations between the items and the containers imposing certain constraints on the placement of items. A finite interactive algorithm is presented whose solution performance is improved by the a priori estimates of the maximum degree of quality of packing.

The four relations mentioned above reflect respectively the degree of mutual attachment of items, mutual compatibility of items, mutual attachment of an item to a container, and mutual compatibility of an item to a container, resulting in matrices taking values from the interval [0, 1]. Containers must be filled with respect to certain conditions e.g. sufficient degree of filling factor, so that the consistency degree of the final packing is maximized and a certain classical total indicator/measure, such as volume or weight, is minimized for the items placed in the reserve container.

In this paper, we extend both the frame packing models and the fuzzy constraint approach of Nasibov (2004), by considering the overall quality of packing in a sequence of frames. We introduce fuzziness in item areas (burst sizes) to be packed within frames to allow partial packing among frames. We assume triangular numbers for the area values to represent minimum admissible service rate and over allocation limit. Hence, by reducing the size of an item, more items can be packed in that frame increasing the capacity utilization. The remainder of the reduced item can be considered in the next frames of the sequence. All item area sizes are positive integers as in frame packing, which depend on item priorities.

We follow the time basis consideration of the frame packing problem by differentiating between the frames. Therefore, the frames are identical in size but their order is important. We use the fuzzy attachment and compatibility relations within items, and between items and frames defined by Nasibov (2004). An attachment value of 1 between an item and frame specifies that the item must be packed in that frame. In contrast, a compatibility value of 0 will indicate the item shouldn't be placed in that frame.

The objective of our model is to maximize the overall quality level defined by the area usage and the attachment and compatibility relations. Denoting the frames by F_i, and items by T_j, a precise statement of our problem is as follows:

(1) There is an ordered sequence of frames F_i with fixed rectangular sizes of width W and height H, $i \in I \cup \{m+1\}$, where $I = \{1...m\}$. F_{m+1} is defined as the reserve frame. Every frame has an area A equal to $W \times H$, where W, H are positive integers;

(2) There is a set of items T_j, where $j \in J = \{1...n\}$ with triangular number area values $\tilde{A}_j = (a_{j1}, a_{j2}, a_{j3})$ where a_{j1}, a_{j2}, a_{j3} are positive integers;

(3) Each item T_j has integer width $w_{j,p}$ and height $h_{j,p}$ combinations such that:
   a. $w_{j,p} \le W$, $h_{j,p} \le H$ for all j and $p \in P_j = \{1...p_j\}$ where p_j is the number of possible width-height pairs for item T_j;
   b. $a_{j1} \le w_{j,p} \times h_{j,p} \le a_{j3}$, for all j, p.

(4) There are four fuzzy attachment and compatibility relations within items, and between items and frames denoted as follows:
   a. $R_1 = \|R_1(T_j, T_k)\|$ is the symmetric and reflexive relation indicating the mutual attachment degree of items T_j and T_k,
   b. $R_2 = \|R_2(T_j, T_k)\|$ is the symmetric and reflexive relation indicating the mutual compatibility degree of items T_j and T_k,
   c. $R_3 = \|R_3(F_i, T_j)\|$ is the relation indicating the mutual attachment degree of frame F_i and item T_j,
   d. $R_4 = \|R_4(F_i, T_j)\|$ is the relation indicating the mutual compatibility degree of frame F_i and item T_j,

   where $i \in I = \{1...m\}$, j, $k \in J = \{1,..,n\}$. The matrices representing these relations will be generated with respect to service parameters, deadlines and item priorities.

(5) The overall quality level of frame sequence packing will use the following degrees for a frame F_i derived by the relations in (4):
   a. Consistency of separation of items placed in the frame,
   b. Mutual compatibility of items placed in the frame,
   c. Consistency of separation of items from the frame,
   d. Mutual compatibility of items placed in the frame and the frame itself.

As for the fuzziness of item areas, let's assume an item T_1's area is defined by the triangular fuzzy number $\tilde{A}_1 = (32, 40, 44)$. Some possible placements of the item T_1 in a frame F_i (W=12, H=13) are depicted in Fig. 2. T_{1a}, T_{1b} and T_{1c} represent different orientations and shapes of the same item T_1 with ideal area of 40 units, respectively with sizes 4x10, 10x4 and 7x6. Assuming the admissible lower bound for the area of item T_1 given by $\tilde{A}_1$, $T_{1a,f1}$, $T_{1b,f1}$ and $T_{1c,f1}$ are respectively some of the fuzzy versions with sizes 8x4, 4x8 and 7x5. In this example T_{1c} can be said to have an over allocation of 2 units corresponding to a 0.5 quality level, whereas $T_{1c,f1}$ with area of 35 units is one other version with quality level 0.375, all implied by $\tilde{A}_1$.

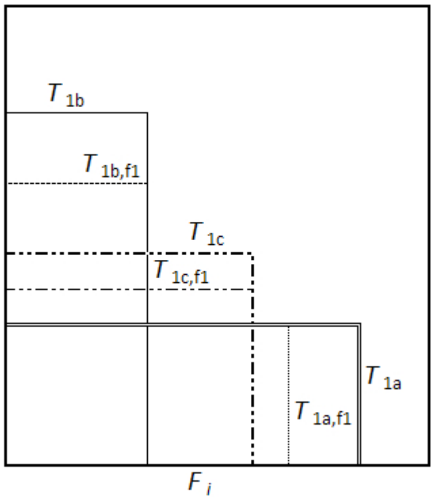

Fig. 2. Possible placements of a variable-sized item with a fuzzy area

We intend to develop a packing mechanism through a sequence of frames. Length of the sequence and the items to be packed will be determined in accordance with the service parameters like user demand quantity, priority and deadline. Next, the fuzzy areas of the items are defined accordingly. Utilizing α-cuts for Aj and heuristics from the literature, we plan to generate and evaluate all possible packing combinations to design new fuzzy relations between item areas and packing algorithms used. For applying the typical bin packing heuristics into our fuzzy approach, we will employ one of the ranking and defuzzification techniques proposed in the literature (Dubois and Prade, 1983; Fortemps and Roubens, 1996).

Besides maximizing the overall quality level described above, common measures like area utilization, height of the packing and over allocation are to be investigated. In a way, our model aims at both satisfying QoS-like service demand constraints in a horizon and to reach efficiency within distinct frame packings.

3. Conclusion and future research

In this paper, we present a general and flexible two-dimensional packing perspective in telecommunications area. Instead of a single objective such as minimization of the height or the maximization of utilization, we apply a quality definition using fuzzy constraints and quantities, integrating different objectives such as QoS priorities.

Using the new model, we intend to simulate different service demand conditions corresponding to different fuzzy area values of the items. Regarding these areas, for different levels of packing quality, all possible packings will be generated. The existing approaches in literature for two-dimensional bin packing or strip packing problems will be adapted for obtaining efficient and high quality solutions. These results can be used to define fuzzy rules for specific area size intervals and specific item priorities. Furthermore, a flexible heuristic selection mechanism may be designed depending on these area and priority parameters.

As another future research direction, we intend to study the version of our model allowing burst compaction, which is the process of combining smaller rectangles into larger rectangles by applying fuzzy clustering techniques.

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