FIRST AND SECOND ORDER SENSITIVITY ANALYSES OF THE ULTIMATE RESISTANCE OF STEEL PLANE FRAMES TO IMPERFECTIONS

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Abstract. The article is aimed at the statistical and sensitivity analysis of the ultimate limit state of steel frames. Load–carrying capacity is calculated for steel plane frames with columns loaded in tension or compression. The Monte Carlo based procedures are used for statistical analysis and for computing the full set of first order and second order sensitivity indices of the model. System imperfections of the frame with compressed columns are considered in the formatively identical to the first eigen mode of buckling. The geometrical nonlinear finite element solution providing numerical result per run was employed. Input random imperfections, which are most deserving of further analysis or measurements, are identified by means of the sensitivity and statistical analysis. Statistical analysis is used to study the influence of the number of columns on the change of the mean value and standard deviation of the load–carrying capacity of frames with tensed columns. Sensitivity analysis is used to identify the dominant input random variables and their higher order interaction effects. The dependence between sensitivity indices and column height is analysed in frames with compressed columns. The sensitivity index of system imperfections has a convex shape. Frame height for which the load–carrying capacity is the most sensitive to the change of system imperfections is determined.

Keywords: structure, frame, steel, strut, imperfection, sensitivity analysis, reliability.

Introduction

The theory of structural reliability was brought to light after Streleckij (Streleckij 1947) and Freudhental (Freudhental 1956) listed the classical theory in the form of two random variables:

\[ R \geq F \] (1)

Where \( R \) represents the ability of the structure to resist external effects, represented here by the symbol \( F \). The interpretation and practical utilization of relation (1) has undergone some development, the most significant stages are presented by the theories of limit states and of allowable stress design.

In the limit state methods the analysis of reliability of structural systems is aimed at safety and serviceability assessment. Determination of dimensions according to the EUROCODE standards emanate from procedures, which are valid for all structures and are based on the comparison of the load effect on one hand and the allowed structural resistance on the other hand. Let us consider in (1) for e.g. the value of allowed deformation on the left side and the value of the deformation from loading on the right side. The validity of the equation (1) can be extended to the field of the evaluation of the serviceability limit state.

Possibilities of choice of analysis types, determination and definition of input variables and also the definition of corresponding limit states of different types of structures remain a topic of discussion. Steel structures are composed of thin members and hence the problem of stability can prove to be one of the most important constituents of the safety; see e.g. (Fukumoto 1997; Kotelko 2007; Kala et al. 2009a). On the other hand the effects of creep, shrinkage and ageing effects can be observed in concrete structures; see e.g. (Gribniak et al. 2007 and 2008; Karmazinová et al. 2009). From the point of view of failure manifestation and reliability, it is possible to point out a certain exceptionality of foundations and geotechnical problems, see e.g. (Juozapaitis et al. 2008; Amšiejus et al. 2009).

Attainment of limit state (generally, occurrence of failure) cannot be absolutely eliminated due to technical and economical reasons and is specific to structural type. The focus of the theory of reliability of building structures is primarily the limit states of cross sections, elements or structures as a whole. Attainment of limit state is generally a random event, which is studied in the the-
ory of reliability by means of the probability theory and mathematical statistics.

The general principles for the reliability of different structures are given by the international standard ISO 2394. The normative principles and application rules of EUROCODES are listed in standard EN1990. The direct assessment of equation (1) is enabled by fully probabilistic methods of structural design, which provide an exact approach to the reliability theory. They are however not usable for practical evaluation of reliability.

In general, probabilistic methods of reliability assessment are utilizable only in cases when mathematical models for the assessment of reliability and sufficient statistical data on input variables of these models are available. Dominant variables can be determined using sensitivity analysis, see (Saltelli et al. 2004; Saltelli et al. 2010). One of the most important characteristics in the reliability assessment of steel structures is the variance of the structural load–carrying capacity, which is given primarily by the quality of production.

Relatively sufficient information on material and geometrical characteristics of mass produced members of steel structures is available in comparison to other buildings. The basic indicators of production quality include the yield strength, tensile strength and ductility, which have been under long term statistical evaluation within the framework of uncommercially aimed research programmes, see e.g. (Melcher et al. 2004; Strauss et al. 2006; Simões da Silva et al. 2009).

The answer to the question, which material and geometric characteristics and imperfections have the greatest influence on the load–carrying capacity can be answered using statistical and sensitivity analysis, which is the focus of this article.

**Problem formulation**

Two variants of frame loading are analyzed in the paper. The load–carrying capacity of a steel plane frame with columns under tension is analyzed in the first variant (see Fig 1), whilst the second variant presents the analysis of the load–carrying capacity of steel plane frame with columns under compression (see Fig 2).

![Fig 1. Steel plane frame with columns under tension](image1)

![Fig 2. Steel plane frame with columns under compression](image2)

Each load–carrying column in the frame depicted in Fig 1 must have a higher load–carrying capacity than the effect arising due to loading F. The load–carrying capacity of each column is a statistically independent random variable determined as a product of the cross–sectional area and the yield strength. The load–carrying capacity of the frame is given as the smallest of the load–carrying capacities of the individual columns. The columns consist of IPE220 profiles. 8 frames with number of columns ranging from 2 to 9 were analysed. The random load–carrying capacity is dependent on the random cross–sectional area and the random yield strength, which are variables that are known from experimental research (Melcher et al. 2004). The aim of the study is the analysis of the influence of the number of columns on the random load–carrying capacity of the frame. The mathematical model for the analysis of the load–carrying capacity of the first frame is obtained analytically.

The geometrical nonlinear finite element solution of the load–carrying capacity must be employed in the case of the second loading variant. The geometrical nonlinear solution was elaborated and programmed by the author of the paper (Kala 2005). The load–carrying capacity of Variant 2 was solved by the nonlinear Euler incremental method combined with the Newton–Raphson method. The frame geometry was meshed by beam elements with initial curvature in the form of a parabola of the 3rd degree. The first criterion (i) for the load–carrying capacity is a loading at which flange plastification is initiated. The second criterion (ii) for the load–carrying capacity is represented by loading corresponding to decrease of the tangential stiffness determinant to zero. The criterion (ii) is applied sporadically, when the random realizations \( e_0 \) are very close to zero, and simultaneously the yield stress realizations of the left and the right web are very high. The ultimate one–parametric loading is defined as the lowest value of load–carrying capacities (i) and (ii).

The aim of the study is the statistical and sensitivity analysis of the load–carrying capacity of frames of Variant 1 and Variant 2. In comparison with the frame with columns under tension, the geometrically nonlinear solution is relatively difficult and time demanding with regard to CPU computer time.

**Input random quantities**

Experimentally obtained cross–section geometry and material characteristics of steel products made by a dominant Czech producer (Melcher et al. 2004) were utilized for the solved problem. In Variant 1 the load–carrying capacity of each column is determined by the yield strength \( f_{yi} \) and cross–sectional area \( A_i \) of profile...
IPE220. The yield strength of steel hot–rolled beam S235 has a mean value of 297.3 MPa and standard (std.) deviation 16.8 MPa. Cross–sectional area has mean value 3293 mm² and std. deviation 102.66 mm². Gauss probability density functions (pdfs) were used.

In Variant 2 the columns of the frame with initial imperfection are loaded by a combination of compression and bending. The determining factors of the load–carrying capacity are the yield strength $f_y$, sectional areas, and second moments of area, Young’s modulus $E$ and system imperfection $e_0$. Statistical characteristics sectional height $h$, sectional width $b$, web thickness $t_w$, flange thickness $t_f$ of profiles IPE 270 and IPE 360 were considered according to (Melcher et al. 2004). For non–measured quantities (e.g., Young’s modulus), the study was based on data obtained from technical literature; for example, statistical characteristics of Young’s modulus are given in (Fukumoto et al. 1976; Soares 1988; Cardoso et al. 2008). The system imperfection $e_0$ (see Fig 2) was considered according to the first buckling mode. The Gauss probability density function (pdf) with mean value $m_{e_0} = 0$ mm, and std. deviation, $S_{e_0} = h/1000$ were assumed for random quantity $e_0$. All the input characteristics are given synoptically in Table 1.

<table>
<thead>
<tr>
<th>No.</th>
<th>Member</th>
<th>Symbol</th>
<th>Mean value</th>
<th>Std. deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Left Column</td>
<td>$h_1$</td>
<td>270.27 mm</td>
<td>1.196 mm</td>
</tr>
<tr>
<td>2.</td>
<td></td>
<td>$b_1$</td>
<td>136.82 mm</td>
<td>1.341 mm</td>
</tr>
<tr>
<td>3.</td>
<td></td>
<td>$t_{w1}$</td>
<td>6.963 mm</td>
<td>0.277 mm</td>
</tr>
<tr>
<td>4.</td>
<td></td>
<td>$t_{f1}$</td>
<td>10.126 mm</td>
<td>0.466 mm</td>
</tr>
<tr>
<td>5.</td>
<td></td>
<td>$E_1$</td>
<td>210 GPa</td>
<td>12.6 GPa</td>
</tr>
<tr>
<td>6.</td>
<td></td>
<td>$f_{y1}$</td>
<td>297.3 MPa</td>
<td>16.8 MPa</td>
</tr>
<tr>
<td>7.</td>
<td></td>
<td>$h_0$</td>
<td>360.36 mm</td>
<td>1.595 mm</td>
</tr>
<tr>
<td>8.</td>
<td></td>
<td>$b_0$</td>
<td>172.3 mm</td>
<td>1.689 mm</td>
</tr>
<tr>
<td>9.</td>
<td></td>
<td>$t_{w0}$</td>
<td>8.44 mm</td>
<td>0.335 mm</td>
</tr>
<tr>
<td>10.</td>
<td></td>
<td>$t_{f0}$</td>
<td>12.611 mm</td>
<td>0.582 mm</td>
</tr>
<tr>
<td>11.</td>
<td></td>
<td>$E_0$</td>
<td>210 GPa</td>
<td>12.6 GPa</td>
</tr>
<tr>
<td>12.</td>
<td></td>
<td>$f_{y0}$</td>
<td>297.3 MPa</td>
<td>16.8 MPa</td>
</tr>
<tr>
<td>13.</td>
<td>Right Column</td>
<td>$h_2$</td>
<td>270.27 mm</td>
<td>1.196 mm</td>
</tr>
<tr>
<td>14.</td>
<td></td>
<td>$b_2$</td>
<td>136.82 mm</td>
<td>1.341 mm</td>
</tr>
<tr>
<td>15.</td>
<td></td>
<td>$t_{w2}$</td>
<td>6.963 mm</td>
<td>0.277 mm</td>
</tr>
<tr>
<td>16.</td>
<td></td>
<td>$t_{f2}$</td>
<td>10.126 mm</td>
<td>0.466 mm</td>
</tr>
<tr>
<td>17.</td>
<td></td>
<td>$E_2$</td>
<td>210 GPa</td>
<td>12.6 GPa</td>
</tr>
<tr>
<td>18.</td>
<td></td>
<td>$f_{y2}$</td>
<td>297.3 MPa</td>
<td>16.8 MPa</td>
</tr>
<tr>
<td>19.</td>
<td>System</td>
<td>$e_0$</td>
<td>0</td>
<td>$h/1000$</td>
</tr>
</tbody>
</table>

* Histogram, ** Gauss pdf

The task of uncertainty analysis is to determine the pdf of the load–carrying capacity (model output), if the pdfs of the material and geometrical (model inputs) parameters are known.

Statistical analysis of Variant 1

Statistical analysis was performed by the Monte Carlo method for 100 000 simulation runs. Change in the mean value and std. deviation of the load–carrying capacity of the frame in Fig 1 can be clearly seen in the Gauss pdf. Increase in the number of columns brings about decrease in mean value and also in std. deviation, see Fig 3. The maximal difference between mean values is approximately 6 %. The std. deviation of the frame with two columns is higher by approximately 41% in comparison with the frame with nine columns.

**Fig 3. Pdfs of load–carrying capacity – Variant 1**

Kolmogorov–Smirnov, Chi–Square and Anderson–Darling distribution tests were performed using the Statrel 3.1 programme, which didn’t provide an unequivocal answer to the question of which pdf is most suitable for the approximation. Due to this reason the Gauss and Hermite pdfs, which presented themselves as the most suitable, were considered.

**Fig 4. Probability of load–carrying capacity – Variant 1**

From the point of view of reliability, the design value, which is obtained according to standard EN1990 as the 0.1 percentile is important. The behaviour of the probabilities (see Fig 4) is obtained by the integration of pdfs from Fig 3. The detailed distribution plot of probability from Fig 4 for design probability of 0.001 is depicted in Fig 5. The maximal difference between design values is approximately 1.4 %, see Fig 5. If we consider the differences between mean values and std. deviations...
described in the previous paragraph, we can say that the difference of 1.4 % is relatively small. Let us also note that the design load–carrying capacity according to EUROCODE 3 is equal to 784 kN, i.e. to the design load–carrying capacity of one column.

The Gaussian pdf is dependent only on the mean value and std. deviation. The std. skewness decreases and std. kurtosis increases with increasing number of columns. The influence of skewness and kurtosis can be taken into account using the so–called the Hermite pdf, which is the Gaussian pdf multiplied by the Hermite polynom with respected skewness and kurtosis (program Statrel 3.10). Comparison of the Gaussian pdfs with the Hermite pdfs is depicted in Fig 6. Probabilities evaluated by the integration of the Hermite pdfs from Fig 6 are depicted in Fig 7. Detail of distribution plots of probabilities from Fig 7 for design probability of 0.001 are illustrated in Fig 8. The maximum difference between design values is approximately 3.6 %, all values are less (safer) than the design load–carrying capacity 784 kN evaluated according to EUROCODE 3.

Differences between density values in Fig 8 are greater than the difference in Fig 5. It generally holds that the value of the 0.1 percentile increases with increasing skewness and that with decreasing kurtosis the value of the 0.1 percentile decreases.

Differences between statistical moments have an influence on the reliability of steel structures, which is generally defined by the probability that the reliability equation (1) is not fulfilled. The reliability of systems comprised of more members is a function of the reliabilities of the individual members and the correlation between the possible methods of failure. The present understanding of reliability defines the reliability of the whole system as the reliability of the weakest member. In the presented example the probability of failure of the frame is generally greater than the probability that failure will occur in one column. It is the case of a series system in which all members must be functional. Let us assume statistical independence, then for small failure probabilities (target values are of the order 10⁻⁵) the failure probability of the whole system is approximately equal to the sum of the partial probabilities.

The opposite of the series system is the parallel system, for which all members must fail in order for the whole structure to fail. In the event of statistical independence of individual members, the failure probability of the whole system is equal to the product of the failure probabilities of the individual members. Generally structures exist as a combination of both systems.
**Statistical analysis of Variant 2**

In the second variant the influence of change in column length $h$ on the statistical characteristics of the load–carrying capacity evaluated for a frame with initial imperfections using the geometric nonlinear solution is investigated. 100 000 simulation runs of the Monte Carlo method were used. The analysis of the influence of the column length on the std. deviation of load–carrying capacity is depicted in Fig 9. The maximum of std. deviation 106.3 kN occurs for system length $h = 8.7$ m (buckling length $L_{cr} = 9.086$ m, nondimensional slenderness according to EUROCODE 3 is $\lambda = 0.86$). The top of the std. deviation curve was obtained by the approximation of three points at the peak of the curve in Fig 9 by a quadratic parabola. The results in Fig 9 provide new information, which may be used in the determination of failure probability in the probabilistic assessment of reliability.

**Fig 9. Std. dev. of load–carrying capacity – Variant 2**

Mean value of load–carrying capacity decreases with increasing values of $h$, the decreasing trend is observed also in the design load–carrying capacity evaluated as the 0.1 percentile, see Fig 10. The distribution plots of the Gaussian and Hermite pdfs differ the most for those values of $h$, for which high values of std. deviations were obtained in Fig 9.

**Fig 10. Mean and design values – Variant 2**

The design value of load–carrying capacity evaluated according to EN1990 as the 0.1 percentile is lower (safer) than the mean value. The design value may be determined according to EUROCODE 3 by using the stability solution with buckling length (see Fig 10) or the geometric nonlinear solution with equivalent geometric imperfections. In the event that sufficient input data is available for the statistical analysis according to EN1990, it is possible to verify the reliability indicators of standard EUROCODE 3. The most relevant data are obtained from a high number of experiments. If these data are not available, it is necessary to distinguish between aleatory (stochastic) and epistemic (state–of–knowledge) uncertainties (Kala 2007; Kala 2008; Vaidogas and Juocevičius 2009).

Difference between the mean and design values in Fig 10 are to a certain degree influenced by the plot of the std. deviation of the load–carrying capacity. The variability of load–carrying capacity (output) is dependent on the variability of material and geometrical characteristics (inputs). One of the important tasks of the probabilistic analysis of reliability is the identification of input random variables that have a dominant influence on the variance of output and reliability.

**Sensitivity analysis**

Sensitivity analysis is the study of how the variation in the output of a model (numerical or otherwise) can be apportioned, qualitatively or quantitatively, on different sources of variation, and of how the given model depends upon the information fed into it (Saltelli et al. 2004). Within the scope of modelling, the notion “sensitivity analysis” has different meaning to different people, see, e.g., (Yang 2007; Kadid 2008; Kotelko et al. 2008; Mang et al. 2009; Melcher et al. 2009; Chow and Chow 2009). Other examples as to why we perform sensitivity analysis are listed in (Kala and Kala 2009). With regard to the limit states of structures, the resistance (or deflection) is frequently considered to be the output variable.

The sensitivity analysis can be generally divided into two groups: (i) deterministic sensitivity analysis and (ii) stochastic sensitivity analysis. Stochastic methods are based upon various assumptions, and it is difficult to compare the results. However, each of the methods has its informative capability of a different kind. These are briefly as follows: (i) the method based upon the observation of the correlation, (ii) Fourier amplitude sensitivity tests (Cukier et al. 1973) and (iii) variance–based methods (Sobol’ 1993). The variance–based techniques are sometimes called ANOVA techniques for ANalysis Of VAriance (Arwade et al. 2010). This paper is devoted to (iii).

**Sobol’s sensitivity analysis**

The concept of sensitivity analysis enabling the analysis of the influence of arbitrary subgroups of input quantities on the monitored output was worked out by the Russian mathematician Ilja M. Sobol (Sobol 1993). The concept of Sobol’s sensitivity analysis was worked out in detail and published along with a number of examples from a number of scientific fields for e.g. in (Saltelli et al. 2004). The case with statistically independent input random variables $X_i$ (input imperfections) was studied. The
Sobol’s first order sensitivity indices may be written in the form:

\[ S_i = \frac{V(E[Y|X_i])}{V(Y)} \]  

(2)

\( S_i \) measures the first order (e.g. additive) effect of \( X_i \) on the model output \( Y \). Sobol proposed an alternate definition \( S_1 = corr(Y, E[Y|X_i]) \) correlation between output \( Y \) and the conditional arithmetical mean \( E[Y|X_i] \).

The second order sensitivity index \( S_{ij} \) is the interaction term (3) between factors \( X_i, X_j \). It captures that part of the response of \( Y \) to \( X_i, X_j \) that cannot be written as a superposition of effects separately due to \( X_i \) and \( X_j \).

\[ S_{ij} = \frac{V(E[Y|X_i, X_j])}{V(Y)} - S_i - S_j \]  

(3)

Other Sobol’ sensitivity indices enabling the quantification of higher order interactions may be expressed similarly.

\[ \sum_i S_i + \sum_{i<j} S_{ij} + \sum_i \sum_{j<k} S_{ijk} + \ldots + S_{123...M} = 1 \]  

(4)

The number of members in (4) is \( 2^M-1 \), i.e. for \( M=3 \) we obtain 7 sensitivity indices \( S_1, S_2, S_3, S_{12}, S_{23}, S_{13}, S_{123} \); for \( M = 10 \) we obtain 1023 sensitivity indices, which is excessively large for practical usage. The main limitation in the determination of all members of (4) is the computational demand.

A study (Kala 2009) describing the influence of the imperfections on the load–carrying capacity of a steel strut was worked out in the field of the stability of steel structures. Higher order interaction effects also have an influence on the monitored output in systems consisting of more members. Sensitivity analysis is applied to the study of the influence of initial imperfections (input values) on the load–carrying capacity (output value) in the presented article. The CPU demanding numerical solutions were ensured by applying the modern computer technology (CPU Intel Extreme Core i7–975 Box).

Possible rationales for sensitivity analysis are:

Identification of critical or otherwise interesting regions in the set of the input imperfections. Identification of imperfections which interact and which may thus generate extreme load–carrying capacities. Official guidelines insist on the importance of taking factor interactions into account (Saltelli et al. 2010). This is important, for example, in structural reliability.

Analysis of which input random imperfection is most deserving of further analysis or measurements. These input imperfections should be measured very accurately even at the expense of those imperfections for which sensitivity is small.

Prior to parameter estimation, to help set up the (actual or numerical) experiment in those conditions in which the sensitivity of the output to the imperfections to be estimated is the greatest.

Further possible motivations for sensitivity analysis are listed e.g. in (Saltelli et al. 2004).

**Sensitivity analysis – Variant 1**

The sensitivity indices were evaluated utilizing the Monte Carlo method. The conditional random arithmetical mean \( E[Y|X_i] \) was evaluated for 100 000 simulation runs; the variance \( V(E[Y|X_i]) \) was calculated for 100 000 simulation runs, as well. The variance \( V(Y) \) of load–carrying capacity was calculated under the assumption that all the input imperfections are considered to be random ones; 1 000 000 runs were applied, as well. The second–order sensitivity indices \( S_{ij} \) were calculated analogously. Interactions of all further higher orders are grouped in a single term.

It is apparent from Fig 11 that the variance of yield strengths of each column has a dominant effect on the variance of load–carrying capacity. The interaction between yield strengths of the left and right columns is also significant. The value of the variance of load–carrying capacity is further influenced by the variance of cross–sectional areas.

![Fig 11. Sensitivity analysis – 2 columns](image)

Difference 1-\( \sum S_i \) is an indicator of the presence of interactions in the model. It is apparent from Fig 11 to Fig 18, that with increasing number of columns decreases the the sum of first–order sensitivity indices, which is lucidly depicted in Fig 19.

Results in Fig 18 show that the sum of first order sensitivity indices has the greatest value followed by the sum of the second order indices. The sum of sensitivity indices of third and higher orders has a relatively small value despite the relatively high number of variables, which could generate higher order interaction effects (frame has 9 columns).
Fig 12. Sensitivity analysis – 3 columns

Fig 13. Sensitivity analysis – 4 columns

Fig 14. Sensitivity analysis – 5 columns

Fig 15. Sensitivity analysis – 6 columns

Fig 16. Sensitivity analysis – 7 columns

Fig 17. Sensitivity analysis – 8 columns
The influence of the first order sensitivity indices of yield strength and cross sectional area is depicted in Fig 20. This influence rises approximately linearly with increasing number of columns. This also points at the increasing significance of the yield strength variance.

The influence of the second order sensitivity index of yield strength to the sum of second order sensitivity indices decreases with increasing column numbers, because the yield strength iterates with a higher number of variables. Higher order interactions are the cause of the decreasing std. deviation of load–carrying capacity.

Sensitivity analysis – Variant 2

The sensitivity indices were evaluated utilizing the Monte Carlo method. In comparison with Variant 1 the analysis is demanding on CPU time per run. $E(Y|X_i)$ was evaluated for 30 000 simulation runs; the variance $V(E(Y|X_i))$ was calculated for 30 000 simulation runs; the variance $V(Y)$ was calculated for 200 000 simulation runs, as well. The sum of first order and second order sensitivity indices is greater than 0.96 for the frame with compressed columns. Due to this reason and the reason of no modest CPU time per run, the third order and higher order sensitivity indices were not analysed (interactions of all further higher orders are grouped in a single term). The dependence of first order sensitivity indices on the column height $h$ of the frame is depicted in Fig 21.

Results of the sensitivity analysis are presented in Fig 21. Due to the fact that the frame, load action and boundary conditions are symmetrical, the values of sensitivity coefficients of both left and right columns are the identical, i.e., they are depicted by a single curve only.

If the frame height $h < 3.2$ m, then the load–carrying capacity variance is most influenced by the yield stress variance of the left and the right column. The first order sensitivity indices $f_{y1}$ ($f_{y2}$) are dominant. The second order effect between $f_{y1} − f_{y2}$ has the most effect on the sum of all second order sensitivity indices. If the frame height $h > 3.2$ m, then the system imperfection $e_0$ is dominant. The maximum effect $S_{e0} = 0.9$ was calculated for the frame with height $h = 9.3$ m (buckling length $L_{cr} = 9.66$m). The comparison of the behaviour of $S_{e0}$ in Fig 21 with Fig 9 is very interesting. More general summaries would require further analyses.

If the frame height $h > 9.3$ m, then the load–carrying capacity is almost insensitive to the changes of yield stress in both columns. With increasing height, the load–carrying capacity approaches the Euler critical force, and therefore it is more sensitive to changes of Young’s module values and of flange thickness in both columns $E_1, E_2, t_{f1}, t_{f2}$.

Conclusion

The statistical and sensitivity analysis of two typical frame structures, i.e. frame with tensed columns – Vari-
Statistical analysis of Variant 1 showed that the mean value and std. deviation of load–carrying capacity decreases with increasing number of columns, see Figs 3, 4, 6, 7. The influence of these changes on the design load–carrying capacity is antagonistic, with increasing column numbers the design value calculated according to EN1990 as 0.1 percentile changes slightly, see Figs 5, 8.

Sensitivity analysis proved that the higher order interaction effects increase with increasing number of columns, see Fig 11 to Fig 19. The significance of the variance of yield strength to the variance of system load–carrying capacity increases with increasing numbers of columns, see Fig 20.

Statistical analysis of Variant 2 showed that graph of std. deviation vs. system height h has a convex character, see Fig 9. Maximum value 106.3 kN of the std. deviation was obtained for system height h = 8.7 m, which corresponds to Lcr = 9.086 m and nondimensional slenderness according to EUROCODE 3 is \( \lambda = 0.86 \). Mean value decreases the fastest for system height h = 9.8 m, see Fig 10. Design value calculated according to EN1990 as 0.1 percentile has a decreasing trend if the system height h increases, see Fig 10. If h < 8.7 m, this decrease is given by a decreasing mean value and an increasing std. deviation. If h > 8.7 m, then the decreasing mean value, but not the std. deviation, which is decreasing too, contribute to this decrease.

Sensitivity analysis showed that the graph of the first order sensitivity index of system imperfection vs. system height h has a convex character, see Fig 21. System imperfections have a dominant influence on the load–carrying capacity for height h > 9.3 m, which corresponds to Lcr = 9.66 m and \( \lambda = 0.92 \). If h > 9.3 m, then the influence of system imperfections decreases and the the influence of those variables that increase the frame stability increases with increasing frame height. If the frame height (column slenderness) increases further and the load–carrying capacity approaches the Euler buckling load, then those variables that are the input variables for the evaluation of the Euler stability solution with buckling length have a dominant influence on the load–carrying capacity. Acquired results of the sensitivity analysis illustrate that the statistical characteristic of system imperfection \( e_0 \) should be determined with increased accuracy, which is however difficult or practically impossible in heavy service conditions.

Efforts for the implementation of probabilistic methods for reliability assessment bring about a number of new problems. Acquired results of the sensitivity analysis illustrate that the statistical characteristic of initial imperfections should be determined with increased accuracy. One important distinction between Sobol’ and classical sensitivity is that the Sobol’ sensitivity analysis detects interactions of input variables through the second and higher order terms, while classical sensitivity methods give only derivatives with respect to single variables (Arwade et al. 2010). The imperfections which interact and may thus generate extreme values of load–carrying capacity have been identified, in particular, for the frames of Variant 1, for which the load–carrying capacity variance is sensitive to the variances of yield strength of columns.

Obtained results may be utilized in standards for design. With the development of the algorithms for nonlinear optimization problems of structures (Atkočiūnas et al. 2008; Kalanta et al. 2009), these procedures can contribute to a qualitative improvement of the reliability analysis of structures. In order for the mathematical models employed for reliability analysis to provide realistic information on the reliability of real steel structures it is necessary that input random variables are obtained from experimental research on ample samples. Many new measurements of uncommercially aimed research are available for model parameters, and therefore the pdfs or at least the variances of the parameters are known (Kala et al. 2009b).

Input random parameters may be divided into two basic groups (Kala 2005). The first group includes those variables whose statistical characteristics can be positively influenced in production (yield strength, geometric characteristics) and those that are not sufficiently sensitive to changes in production technology (e.g. variability of Young’s modulus E). The first group of variables may be further divided into two subgroups: (i) variables for which mean value and std. deviation can be changed by improvement of production quality (Kala 2005). Examples include Young’s modulus; (ii) variables, the mean value of which cannot be significantly changed, because it should approximately correspond to the nominal value (geometric characteristics of profile dimensions).

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